10th Class 2017

Math (Science)	Group-I	PAPER-II
Time: 2.10 Hours	(Subjective Type)	Max. Marks: 60

(Part-I)

2. Write short answers to any SIX (6) questions: (12)

(i) Write the name of any two methods for solving a quadratic equation.

Ans The name of any two methods for solving a quadratic equation are:

1. Factorization Method.
2. Completing Square Method.

(ii) Solve: $x^2 + 2x - 2 = 0$

Ans Here, a = 1, b = 2, c = –2

We may solve the above equation through quadratic formula, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4 + 8}}{2}$$

$$= \frac{-2 \pm \sqrt{12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}}{2}$$

$$= -1 \pm \sqrt{3}$$

(iii) Evaluate : $(1 - 3w - 3w^2)^5$

Ans Given:

$$(1 - 3w - 3w^2)^5$$

By taking common, we get

$$= [1 - 3(w + w^2)]^5$$

As we know that: $w + w^2 = -1$

$$= [1 - 3(-1)]^5$$
$$= (1 + 3)^5$$
$$= 4^5$$
$$= 1024$$

(iv) Evaluate: $\omega^{37} + \omega^{38} - 5$.

Ans Given:

$$\omega^{37} + \omega^{38} - 5$$
$$= \omega^{36}.\omega + \omega^{36}.\omega^2 - 5$$
$$= \omega^{36}(\omega + \omega^2) - 5$$
$$= (\omega^3)^{12}(-1) - 5$$
$$= -1 - 5$$
$$= -6$$

(v) Without solving find the sum and the product of roots of quadratic equation: $3x^2 + 7x - 11 = 0$.

Ans Here, $a = 3, b = 7, c = -11$

Sum of the roots:

$$S = \alpha + \beta = \frac{-b}{a} = \frac{-7}{3}$$

Product of the roots:

$$P = \alpha\beta = \frac{c}{a} = \frac{-11}{3}$$

(vi) Write the quadratic equation having the roots: $-1, -7$.

Ans Sum of the roots:

$$\alpha + \beta = -1 + (-7) = -8$$

Product of the roots:

$$\alpha\beta = (-1)(-7) = 7$$

Thus the quadratic equation will be:

$$x^2 - Sx + P = 0$$
$$x^2 - (-8)x + 7 = 0$$
$$x^2 + 8x + 7 = 0$$

(vii) Define direct variation.

Ans If two quantities are related in such a way that increase (decrease) in one quantity causes increase

(decrease) in the other quantity, then this variation is called direct variation.

(viii) Find the fourth proportional to 8, 7, 6.

Ans Let the fourth proportional is x:

$$8 : 7 :: 6 : x$$
$$8 \times x = 7 \times 6$$
$$x = \frac{42}{8}$$
$$x = \frac{21}{4}$$

(ix) Find x if 6 : x : : 3 : 5.

Ans

$$x \times 3 = 6 \times 5$$
$$x = \frac{6 \times 5}{3}$$
$$x = \frac{30}{3}$$
$$\boxed{x = 10}$$

3. Write short answers to any SIX (6) questions: (12)

(i) Define a rational fraction.

Ans An expression of the form $\frac{N(x)}{D(x)}$, where N(x) and D(x) are polynomials in x with real coefficients and $D(x) \neq 0$, is called a rational fraction.

(ii) Resolve $\frac{1}{x^2 - 1}$ into partial fraction.

Ans

$$\frac{1}{x^2 - 1} = \frac{1}{(x + 1)(x - 1)}$$
$$\frac{1}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$$
$$1 = \frac{A}{(x + 1)}(x^2 - 1) + \frac{B}{(x - 1)}(x^2 - 1)$$
$$1 = A(x - 1) + B(x + 1) \qquad \text{(i)}$$

Put $x = 1$ in (i)

$$1 = A(1 - 1) + B(1 + 1)$$

$1 = 0 + 2B$

$2B = 1$

$$\boxed{B = \frac{1}{2}}$$

Similarly, put $x = -1$ in (i),

$1 = A(-1 - 1) + B(-1 + 1)$

$1 = A(-2) + 0$

$1 = -2A$

$\Rightarrow \quad -2A = 1$

$$\boxed{A = \frac{-1}{2}}$$

Finally, by putting the values in (i), we have

$$\frac{1}{x^2 - 1} = \frac{1}{2(x - 1)} - \frac{1}{2(x + 1)}$$

(iii) Define subset.

Ans If A and B are two sets and every element of A is a member of B, then A is called subset of B.

(iv) If L = {a, b, c}, M = {3, 4}, then find L × M.

Ans $L \times M = \{a, b, c\} \times \{3, 4\}$

$L \times M = \{(a, 3), (a, 4), (b, 3), (b, 4), (c, 3), (c, 4)\}$

(v) Find domain and range of the binary relation, R = {(1, 1), (2, 2), (3, 3), (4, 4)}.

Ans Dom $R = \{1, 2, 3, 4\}$

Range $R = \{1, 2, 3, 4\}$

(vi) If (2a + 5, 3) = (7, b – 4), find a, b.

Ans By comparing the values, we get

$2a + 5 = 7 \quad ; \quad 3 = b - 4$

$2a = 7 - 5 \quad ; \quad 3 + 4 = b$

$2a = 2 \quad ; \quad 7 = b$

$a = \frac{2}{2} \quad ; \Rightarrow \quad \boxed{b = 7}$

$$\boxed{a = 1}$$

Thus, $\{a = 1, b = 7\}$.

(vii) Write two properties of arithmetic mean.

Ans Two properties of arithmetic mean are:

1. Mean is affected by change in origin.
2. Sum of the deviations of the variable X from its mean is always zero.

(viii) Define mode.

Ans Mode is defined as the most frequent value in the data.

(ix) The sugar contents for a random sample of 6 packs of juices of a certain brand are found to be 2.3, 2.7, 2.5, 2.9, 3.1 and 1.9 milligram, find the median.

Ans Arranging the values by increasing order 1.9, 2.3, 2.5, 2.7, 2.9, 3.1.

$$\text{Median} = \frac{1}{2}[\text{size of } (3^{rd} + 4^{th}) \text{ values}]$$

$$= \frac{2.5 + 2.7}{2}$$

$$= 2.6 \text{ Milligram}$$

4. Write short answers to any SIX (6) questions: (12)

(i) Define radian measure of an angle.

Ans The angle subtended at the centre of the circle by an arc, whose length is equal to the radius of the circle is called one Radian.

(ii) Convert 15° to radian.

Ans
$$15^\circ = 15 \times \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{12} \text{ radian}$$

(iii) Find 'r', when $l = 56$ cm, $\theta = 45^\circ$.

Ans $l = 56$ cm, $\theta = 45^\circ$, $r = ?$

By converting the θ into radians,

$$45^\circ = 45 \times \frac{\pi}{180} \text{ radian}$$

$$= \frac{\pi}{4} \text{ radians}$$

We have,

$$l = r\,\theta$$

$$\Rightarrow \quad r = \frac{l}{\theta}$$

$$= \frac{56}{\frac{\pi}{4}}$$

$$= \frac{56 \times 4}{\pi}$$

$$r = 71.27 \text{ cm}$$

(iv) What is meant by zero dimension?

Ans Projection of a vertical line segment $\overline{CD}$ on a line segment $\overline{AB}$ is a point on $\overline{AB}$ which is of zero dimension.

(v) Define chord of a circle.

Ans The joining of any two points on the circumference of the circle is called chord of a circle.

(vi) Define tangent to a circle.

Ans A tangent to a circle is the straight line which touches the circumference at one point only.

(vii) What is meant by sector of a circle?

Ans The sector of a circle is an area bounded by any two radii and the arc intercepted between them.

(viii) Define circumangle.

Ans A circumangle is subtended between any two chords of a circle, having common point on its circumference.

(ix) Define inscribed circle.

Ans A circle which touches the three sides of a triangle internally is known as inscribed circle.

(Part-II)

NOTE: Attempt THREE (3) questions in all. But question No. 9 is Compulsory.

Q.5.(a) Solve the equation by completing square: (4)

$$11x^2 - 34x + 3 = 0$$

Ans

$$11x^2 - 34x = -3$$

$$x^2 - \frac{34}{11}x = \frac{-3}{11}$$

Adding $\left(\frac{17}{11}\right)^2$ on both sides,

$$x^2 - 2(x)\left(\frac{17}{11}\right) + \left(\frac{17}{11}\right)^2 = \frac{-3}{11} + \left(\frac{17}{11}\right)^2$$

$$\left(x - \frac{17}{11}\right)^2 = \frac{-3}{11} + \frac{289}{121}$$

$$= \frac{-33 + 289}{121}$$

$$= \frac{256}{121}$$

Taking square root on both sides, we have

$$x - \frac{17}{11} = \pm\frac{16}{11}$$

$$x = \frac{17}{11} \pm \frac{16}{11}$$

$$x = \frac{17}{11} + \frac{16}{11} \quad ; \quad x = \frac{17}{11} - \frac{16}{11}$$

$$= \frac{17 + 16}{11} \quad ; \quad = \frac{17 - 16}{11}$$

$$= \frac{33}{11} \quad ; \quad x = \frac{1}{11}$$

$$x = 3$$

(b) If α, β are the roots of equation $lx^2 + mx + n = 0$, $(l \neq 0)$, then find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. (4)

Ans $a = l, \quad b = m, \quad c = n$

$$\alpha + \beta = \frac{-b}{a} = \frac{-m}{l}$$

$$\alpha\beta = \frac{c}{a} = \frac{n}{l}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{\left(\frac{-m}{l}\right)^2 - 2\left(\frac{n}{l}\right)}{\left(\frac{n}{l}\right)^2}$$

$$= \frac{\frac{m^2}{l^2} - \frac{2n}{l}}{\frac{n^2}{l^2}}$$

$$= \frac{\frac{m^2 - 2ln}{l^2}}{\frac{n^2}{l^2}}$$

$$= \frac{1}{n^2}(m^2 - 2ln)$$

Q.6.(a) Using theorem of componendo-dividendo find the value of: $\frac{x + 2y}{x - 2y} + \frac{x + 2z}{x - 2z}$ if $x = \frac{4yz}{y + z}$. (4)

Ans

$$x = \frac{4yz}{y + z}$$

$$\frac{x}{2y} = \frac{2z}{y + z}$$

$$\frac{x+2y}{x-2y}=\frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y}=\frac{y+3z}{z-y} \qquad (1)$$

Similarly,

$$\frac{x}{2z}=\frac{2y}{y+z}$$

$$\frac{x+2z}{x-2z}=\frac{2y+y+z}{2y-y-z}$$

$$=\frac{3y+z}{y-z}$$

$$=-\left(\frac{3y+z}{z-y}\right)$$

$$\frac{x+2z}{x-2z}=\frac{-3y-z}{z-y} \qquad (2)$$

From (1) and (2), we have

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{y+3z}{z-y}+\frac{-3y-z}{z-y}$$

$$=\frac{y+3z-3y-z}{z-y}$$

$$=\frac{2z-2y}{z-y}$$

$$=\frac{2(z-y)}{z-y}$$

$$=2$$

(b) Resolve into partial fractions: $\frac{x-11}{(x-4)(x+3)}$. **(4)**

Ans

$$\frac{x-11}{(x-4)(x+3)}=\frac{A}{x-4}+\frac{B}{x+3}$$

$$x-11=A(x+3)+B(x-4) \qquad (i)$$

Put $x=4$, $x=-3$ in (i)

Firstly, $4-11=A(4+3)+B(4-4)$

$$-7=A(7)+0$$

$$\Rightarrow \quad 7A = -7$$

$$\boxed{A = -1}$$

And

$$-3 - 11 = A(-3 + 3) + B(-3 - 4)$$
$$-14 = 0 + B(-7)$$
$$\Rightarrow \quad -7B = -14$$
$$\boxed{B = 2}$$

So,

$$\frac{x - 11}{(x - 4)(x + 3)} = \frac{-1}{x - 4} + \frac{2}{x + 3}$$

Q.7.(a) If U = {1, 2, 3, -----, 10}, A = {1, 3, 5, 7, 9}, B = {1, 4, 7, 10}, then verify that A – B = A ∩ B′. (4)

Ans L.H.S = A – B

= {1, 3, 5, 7, 9} – {1, 4, 7, 10}

= {3, 5, 9} (i)

B′ = U – B

= {1, 2, 3, -----, 10} – {1, 4, 7, 10}

= {2, 3, 5, 6, 8, 9}

R.H.S = A∩B′

= {1, 3, 5, 7, 9} ∩ {2, 3, 5, 6, 8, 9}

= {3, 5, 9} (ii)

So, proved from (i) and (ii),

L.H.S = R.H.S

A – B = A∩B′

(b) Calculate the variance for the data: (4)

10, 8, 9, 7, 5, 12, 8, 6, 8, 2

Ans

X	X^2
10	100
8	64
9	81
7	49
5	25
12	144
8	64

6	36
8	64
2	4
75	631

Here, $\Sigma X = 75$, $\Sigma X^2 = 631$, $n = 10$

$$\text{Variance} = S^2 = \frac{\Sigma X^2}{n} - \left(\frac{\Sigma X}{n}\right)^2$$

$$= \frac{631}{10} - \left(\frac{75}{10}\right)^2$$

$$= 63.1 - 56.25$$

$$S^2 = 6.85$$

Q.8.(a) Prove that: $\sin\theta(\tan\theta + \cot\theta) = \sec\theta$. (4)

Ans L.H.S $= \sin\theta(\tan\theta + \cot\theta)$

$$= \sin\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \sin\theta\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$

$$= \sin\theta\left(\frac{1}{\cos\theta\sin\theta}\right)$$

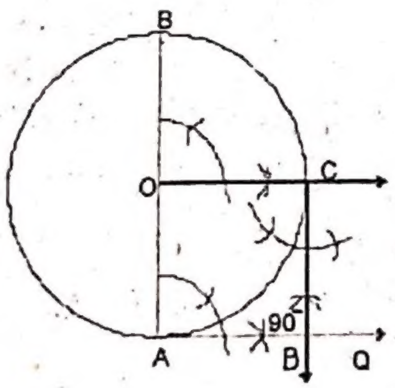
$$= \frac{1}{\cos\theta}$$

$$= \sec\theta$$

= R.H.S Proved

(b) Draw two perpendicular tangents to a circle of radius 3 cm.

Ans



Step of Construction:

Steps:

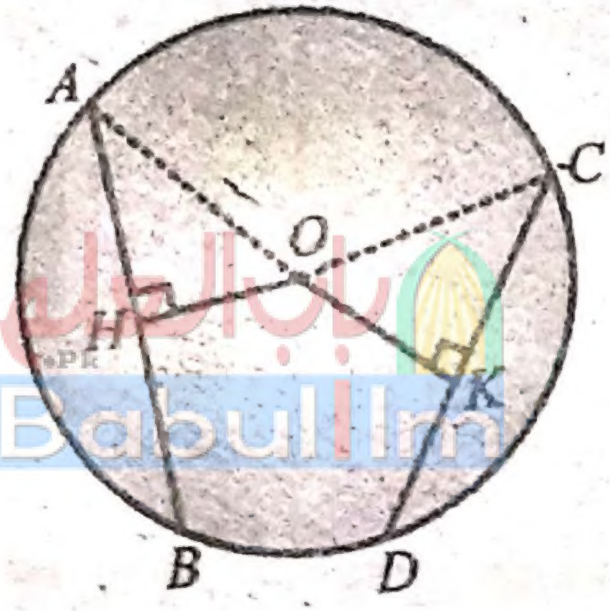
1. Take a point O.
2. Take O as centre and draw circle of radius 3 cm.
3. Take AOB any diameter of the circle.
4. Draw m $\angle$ BOC = 90°, m $\angle$ AOC = 90°.
5. Draw tangents at point A, C. These are $\overrightarrow{CP}$, $\overrightarrow{AQ}$.

Result:

$\overrightarrow{AQ}$, $\overrightarrow{CP}$ are required tangents at point D at 90°.

Q.9. Prove that if two chords of a circle are congruent, then they will be equidistant from the centre. (4)

Ans



Given:

$\overline{AB}$ and $\overline{CD}$ are two equal chords of a circle with centre at O.

So that $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$.

To prove:

$m\overline{OH} = m\overline{OK}$

Construction:

Join O with A and O with C.

So that we have $\angle rt\Delta^s$ OAH and OCK.

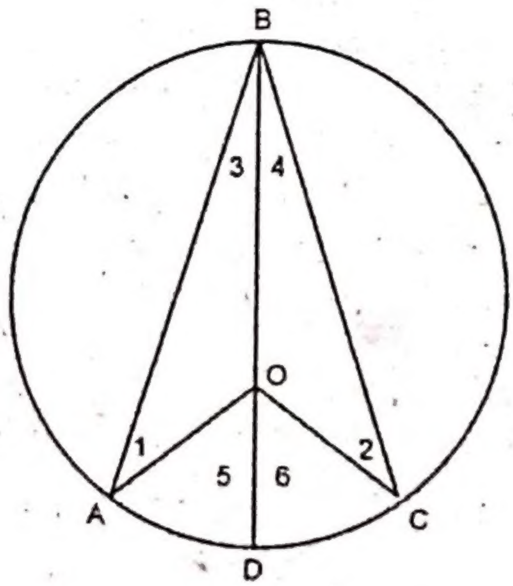
Proof:

Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ By Theorem 3
i.e., $m\overline{AH} = \frac{1}{2} m\overline{AB}$ (i)	
Similarly, $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ By Theorem 3
i.e., $m\overline{CK} = \frac{1}{2} m\overline{CD}$ (ii)	
But $m\overline{AB} = m\overline{CD}$ (iii)	Given
Hence, $m\overline{AH} = m\overline{CK}$ (iv)	Using (i), (ii) & (iii)
Now in $\angle$rt Δ^s $OAH \leftrightarrow OCK$	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp $\overline{OA}$ = hyp $\overline{OC}$	Radii of the same circle
$m\overline{AH} = \overline{CK}$	Already proved in (iv)
$\therefore$ $\Delta OAH \cong \Delta OCK$	H.S postulate
$\Rightarrow$ $m\overline{OH} = m\overline{OK}$	

OR

Prove that the measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Ans



Given:

$\overset{\frown}{AC}$ is an arc of a circle with center O; whereas $\angle AOC$ is the central angle and $\angle ABC$ is circumangle.

To prove:

$m\angle AOC = 2m\angle ABC$

Construction:

Join B with O and produce it to meet the circle at D.

Write angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ as shown in the figure.

Proof:

Statements	Reasons
As $m\angle 1 = m\angle 3$ (i)	Angles opposite to equal sides in ΔOAB.
and $m\angle 2 = m\angle 4$ (ii)	Angles opposite to equal sides in ΔOBC.
Now $m\angle 5 = m\angle 1 + m\angle 3$ (iii)	External angle is the sum of internal opposite angles.
Similarly, $M\angle 6 = m\angle 2 + m\angle 4$ (iv)	
Again $M\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3$ (v)	Using (i) and (iii)
And $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4$ (vi)	Using (ii) and (iv)
$\Rightarrow m\angle 5 + m\angle 6 = 2m\angle 3 + m\angle 4$	Adding (v) and (vi)
$\Rightarrow m\angle AOC = 2(m\angle 3 + m\angle 4)$	
$= 2m\angle ABC$	